The sequence \textbf{A120733} in OEIS is "Number of matrices with nonnegative integer entries and without zero rows or columns such that sum of all entries is equal to \( n \)."

\textbf{Main result:}

\[ A120733(n) \sim \frac{2^{\log(2)} - 2 \cdot n!}{(\log(2))^{2n+2}} \]

\textbf{Proof:}

In OEIS we have a formula

\[ A120733(n) = \frac{1}{n!} \sum_{k=1}^{n} (-1)^{n-k} \cdot S_1(n,k) \cdot A000670(k)^2 \]

where \( S_1(n,k) \) are the Stirling numbers of the first kind and \( A000670 \) are Fubini numbers (number of ordered partitions of \( n \), the ordered Bell numbers)

The sequence \( A000670 \) has an exponential generating function

\[ f(x) = \frac{1}{2 - e^x} \]

with a simple pole at \( r = \log(2) \) and the derivative is

\[ f'(x) = \frac{e^x}{(2 - e^x)^2} \]

Asymptotic is then

\[ A000670(n) \sim \frac{\text{residue}(f, r)}{r^{n+1}} \cdot n! = \frac{f(r)^2}{f'(r) \cdot r^{n+1}} \cdot n! = \frac{n!}{2 \cdot (\log(2))^{n+1}} \]

Now

\[ A120733(n) = \frac{1}{n!} \cdot A000670(n)^2 \cdot \sum_{k=1}^{n} (-1)^{n-k} \cdot S_1(n,k) \cdot \left( \frac{A000670(k)}{A000670(n)} \right)^2 \]

The maximal term in the sum is at the position \( k = n \) (see a graph in the logarithmical scale)

\[ \sum_{k=1}^{n} (-1)^{n-k} \cdot S_1(n,k) \cdot \left( \frac{A000670(k)}{A000670(n)} \right)^2 = 1 - S_1(n, n-1) \cdot \left( \frac{A000670(n-1)}{A000670(n)} \right)^2 + S_1(n, n-2) \cdot \left( \frac{A000670(n-2)}{A000670(n)} \right)^2 - \ldots \]

For fixed \( k \) we have (see H. W. Gould, formula 8.4):

\[ (-1)^k \cdot S_1(n, n-k) \sim \frac{n^{2k}}{2^k k!} \]
Together
\[
\frac{A000670(n-k)}{A000670(n)} \sim \frac{(n-k)!}{n!} \cdot (\log(2))^k \sim \left(\frac{\log(2)}{n}\right)^k
\]

Contribution of all terms in the sum is
\[
(-1)^k \cdot S_i(n, n-k) \cdot \left(\frac{A000670(n-k)}{A000670(n)}\right)^2 \sim \frac{n^{2k}}{2^k \cdot k!} \cdot \left(\frac{\log(2)}{n}\right)^{2k} = \frac{(\log(2))^{2k}}{2^k \cdot k!}
\]

The final asymptotic is
\[
A120733(n) \sim \frac{1}{n!} \cdot \left(\frac{n!}{2 \cdot (\log(2))^{n+1}}\right)^2 \cdot e^{\frac{(\log(2))^2}{2}} = \frac{\log^2(2) - 2 \cdot n!}{(\log(2))^{2n+2}}
\]

**Numerical verification**, the ratio tends to 1:

![Numerical verification graph](image)

Richardson extrapolation, 10 steps, from 100 terms of the sequence. The convergence is very good.

![Richardson extrapolation graph](image)